1. Using $x_{n+1} = -2 - \dfrac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$$x_1 = -2 - \frac{4}{(-2.5)^2} = -2.64 \quad ①$$

$$x_2 = -2 - \frac{4}{(-2.64)^2} = -2.573921028 \quad ①$$

$$x_3 = -2 - \frac{4}{(-2.57...)^2} = -2.603767255$$

①

$x_1 =$ -2.64

$x_2 =$ -2.57

$x_3 =$ -2.60

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

$x_{n+1} = -2 - \dfrac{4}{x_n^2}$. $\therefore$ $x^3 = -2x^2 - 4$. $x^3 + 2x^2 + 4 = 0$.

$x^3 + 2x^2 + 4 = 0$ is a rearrangement of the iterative equation in part (a). ① x_1, x_2 and x_3 are estimations of the solutions to $x^3 + 2x^2 + 4 = 0$. ①

(2)

(Total for Question is 5 marks)

2. (a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

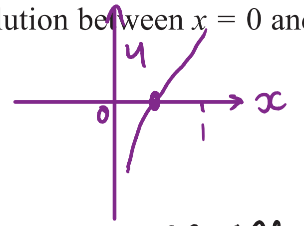
When $x = 0$

$(0)^3 + 7(0) - 5$

$= -5$

When $x = 1$

$(1)^3 + 7(1) - 5$



Therefore, because one of the values is negative and one of the values is positive

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \dfrac{5}{x^2 + 7}$

$x^3 + 7x - 5 = 0$

$x(x^2 + 7) - 5 = 0$

$(+5) \quad (+5)$

$x(x^2 + 7) = 5$

$(\div (x^2 + 7))$

$x = \dfrac{5}{x^2 + 7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \dfrac{5}{x_n^{\,2} + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

$x_1 = \dfrac{5}{x_0^{\,2} + 7}$

$x_1 = \dfrac{5}{1^2 + 7}$

$= 0.625$

$x_2 = \dfrac{5}{x_1^{\,2} + 7}$

$x_2 = \dfrac{5}{(0.625)^2 + 7}$

$= 0.67653277$

$x_3 = \dfrac{5}{x_2^{\,2} + 7}$

$x_3 = \dfrac{5}{(0.67653277)^2 + 7}$

$= 0.670448$

..........0.670448..........

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$,
comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

Answer to part c) = 0.670 448

$(0.670448)^3 + 7(0.670448) - 5 = -0.0054948$

Estimate is accurate because the substitution gives us a value close to 0

(2)

(Total for Question is 9 marks)

3. (a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$x^3 + x = 7$

an x value between 1 and 2 satisfies this equation

$x^3 + x - 7 = 0$ let $y = x^3 + x - 7$

When $x = 1$ $y = 1^3 + 1 - 7 = 1 + 1 - 7 = -5$

$x = 2$ $y = 2^3 + 2 - 7 = 8 + 2 - 7 = 3$ (1)

There is a change in sign (-5 to 3) so there must be a solution between 1 and 2 (1)

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$x^3 + x = 7$

$x^3 = (7 - x)$

$\sqrt[3]{x^3} = \sqrt[3]{7 - x}$

$x = \sqrt[3]{7 - x}$ (1)

(1)

(c) Starting with $x_0 = 2$,
use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$x_1 = \sqrt[3]{7 - x_0}$

$x_1 = \sqrt[3]{7 - 2} = \sqrt[3]{5} = 1.70997\ldots$ (1)

$x_2 = \sqrt[3]{7 - x_1}$

$x_2 = \sqrt[3]{7 - 1.70997\ldots} = 1.74241\ldots$ (1)

$x_3 = \sqrt[3]{7 - x_2}$

$x_3 = \sqrt[3]{7 - 1.74241\ldots} = 1.73884\ldots$

$= 1.74$

Use the exact values stored in your calculator to make sure there is no error in further iterations due to rounding.

1.74 (1)

(3)

(Total for Question is 6 marks)

4. The number of rabbits on a farm at the end of month n is P_n
The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2P_n - 50$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June.

March $n=1$ $P_1 = 200$ -10

April $n=2$ $P_2 = 1.2(200) - 50 = 190$ (1) -12

May $n=3$ $P_3 = 1.2(190) - 50 = 178$ (1) -15

June $n=4$ $P_4 = 1.2(178) - 50 = 163.6$ $\therefore$ 163 rabbits (1)

163

(3)

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

There won't be any rabbits (1)

Since number decreases more + more each year as seen above

(1)

5. A hot air balloon is descending.
The height of the balloon n minutes after it starts to descend is h_n metres.

The height of the balloon $(n + 1)$ minutes after it starts to descend, h_{n+1} metres, is given by

$$h_{n+1} = K \times h_n + 20 \quad \text{where } K \text{ is a constant.}$$

The balloon starts to descend from a height of 1200 metres at 09 15
At 09 16 the height of the balloon is 1040 metres.

Work out the height of the balloon at 09 18

1) find k : $1040 = K \times 1200 + 20$ -20

$\div 1200$ $1020 = k \times 1200$ $\div 1200$

$K = 0.85$

2) $h_{9:17} = 0.85 \times 1040 + 20 = 904$

$h_{9:18} = 0.85 \times 904 + 20 = 788.4\text{ m}$

.... m

(Total for Question is 4 marks)

6. (a) Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1, x_2 and x_3
Start with $x_0 = 2$

$x_0 = 2$

$x_1 = \sqrt[3]{10-2(2)} = \sqrt[3]{10-4} = \sqrt[3]{6} = 1.8171\ldots$ (1)

$x_2 = \sqrt[3]{10-2(1.8171\ldots)} = 1.8533\ldots$ (1)

DON'T ROUND
use all decimal points
by using calculator ANS
button

$x^3 = \sqrt[3]{10-2(1.8533\ldots)} = 1.8462\ldots$ (1)

$x_1 =$ 1.8171...

$x_2 =$ 1.8533...

$x_3 =$ 1.8462...

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

(b) Find the value of a and the value of b.

$x_{n+1} = \sqrt[3]{10-2x_n}$ ← from a)

↓

$x = \sqrt[3]{10-2x} \Rightarrow x^3 = 10-2x$

$\Rightarrow x^3 + 2x + (-10) = 0$

$a =$ 2

$b =$ -10 (1)

(1)